Element of Modern Physics

Model Problem Set

Unit I

1. Find the peak wavelength of the blackbody radiation emitted by the human body when the skin temperature is 35°C. *Ans: 9.41 mm*

2. Lightning produces a maximum air temperature on the order of 10^4 K, whereas a nuclear explosion produces a temperature on the order of 10^7 K. (a) Use Wien's displacement law to find the order of magnitude of the wavelength of the thermally produced photons radiated with greatest intensity by each of these sources. (b) Name the part of the electromagnetic spectrum where you would expect each to radiate most strongly.

Ans: (a)$2.897\times10^{-7}A^o$ (b) lightning: ultraviolet; explosion: X-ray and gamma ray

3. The average threshold of dark-adapted (scotopic) vision is 4.00×10^{-11} W/m^2 at a central wavelength of 500 nm. If light with this intensity and wavelength enters the eye and the pupil is open to its maximum diameter of 8.50 mm, how many photons per second enter the eye? *Ans: 5.71 photons/s*

4. Molybdenum has a work function of 4.20 eV. (a) Find the cutoff wavelength and cutoff frequency for the photoelectric effect. (b) What is the stopping potential if the incident light has a wavelength of 180 nm? *Ans: (a) 295 nm, 1.02 PHz (b) 2.69 V*

5. Suppose that light of total intensity 1μW/ cm^2 falls on a clean iron sample1 cm^2 in area. Assume that the iron sample reflects 96% of the light and that only 3% of the absorbed energy lies in the violet region of the spectrum above the threshold frequency.

a. What intensity is actually available for the photoelectric effect? *Ans: 1.2nW/cm^2*

b. Assuming that all the photons in the violet region have an effective wavelength of 250 nm, how many electrons will be omitted per second? *Ans: 1.5×10^9*

c. Calculate the current in the photo tube in Amperes. *Ans: 2.4×10^{-10} A*

d. If the cut-off frequency is 1.1×10^{15} Hz, find the work function for the iron.

Ans: 4.5eV

e. Find the stopping voltage for iron if photoelectrons are produced by light with wavelength 250 nm. *Ans: 0.46V*

6. A 0.0016 nm photon scatters from a free electron. For what (photon) scattering angle does the recoiling electron have kinetic energy equal to the energy of the scattered photon?

Ans: 70.0°

7. X-rays having an energy of 300 keV undergo Compton scattering from a target. The scattered rays are detected at 37.0° relative to the incident rays. Find (a) the Compton shift at this angle, (b) the energy of the scattered x-ray, and (c) the energy of the recoiling electron.

Ans: (a) 4.89 nm (b) 268 keV (c) 31.8 keV

8. Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1A°, which is the size of the interatomic spacing of atoms in a crystal.

Ans: 151 V

9. An electron of energy 200 eV is passed through a circular hole of radius 10^{-4} cm. What is the uncertainty introduced in the angle of emergence?

Ans: 6×10^{-6} radians = 1sec of arc

10. Given that when light is shone on an aluminum target (work function for Aluminum is 4.08 eV) leads to the emission of electrons with maximum kinetic energy of 5.0 eV, what was the frequency of the light?

Ans: 2.2×10^{15} Hz

11. What should be the kinetic energy of an electron so that its de-Broglie wave length is same as that of X-rays produced in an X-ray tube operating at 30 kV?

Ans: 1.4×10^{-16} J

12. A photon of wavelength 3A° suffers Compton scattering by a free electron originally at rest. If the angle of scattering is 90^0, what is the kinetic energy of recoil electron?

Ans: 33.3 eV

13. The emitter in a photoelectric tube has a threshold of 6000 A^o. Determine the wavelength on the light incident on the tube if the stopping potential for this light is 2.5 V.

Ans: 2713 A^0

14. A proton is confined to space of radius 10^{-14} m. Calculate the minimum uncertainty in its momentum. Also calculate the minimum kinetic energy the proton should have mass of the proton is 1.672×10^{-27} kg and h=6.63×10^{-34} Js.

Ans: 0.53×10^{-20} kg-m/s; 0.0525 MeV

15. A thermal neutron has a speed v at temperature T = 300K and kinetic energy $\frac{m_n v^2}{2} = \frac{3kT}{2}$, Calculate its de-Broglie wavelength. State whether a beam of these neutrons could be diffracted by a crystal, and why? (b) Use Heisenberg's Uncertainty principle to estimate the kinetic energy (in MeV) of a nucleon bound within a nucleus of radius 10^{-15} m.

Ans: 1.37 A^0, Yes (λ comparable with interatomic spacing), 20.5 MeV

16. Certain surface waves in a fluid travel with phase velocity $\sqrt{\frac{b}{\lambda}}$ where b is a constant. Find the group velocity of a packet of surface waves, in terms of phase velocity.

Ans : 1.5 V_{phase}

17. Compute the de-Broglie wavelength of a) 1000 kg automobile travelling at 100m/s, b) 10g bullet travelling at 500 m/s, c) a smoke particle of mass 10^{-9} g moving at 1 cm/s, d) An electron with Kinetic energy 1 eV e) an electron with kinetic energy 100 MeV.

Ans : 6.6×10^{-39} m, 1.3×10^{-34} m, 6.6×10^{-20} m, 1.2 nm, 12 fm

18. The dispersion relation for free relativistic electron waves is given by

$$w(k) = [c^2k^2 + (m_c c^2 / \hbar)^2]^{1/2}$$

Obtain expressions for the phase velocity v_p and group velocity v_g of these waves and show that their product is a constant, independent of k. From your result, what can you conclude about v_g if $v_p > c$?

Unit II

1. The speed of a bullet (m=50g) and a speed of an electron ($m=9.1 \times 10^{-28}$ g) are measured to be same, namely 300 m/s, with an uncertainty of 0.01 %. With what fundamental accuracy could we have located the position of each, if the position is measured simultaneously with the speed in the same experiment? *Ans : 7×10^{-32}m*

2. The nucleon is confined to a nucleus of radius 5×10^{-15} m. Calculate the minimum uncertainty in the momentum of the nucleon. Also calculate the minimum kinetic energy of the nucleon. *Ans: 8.25 MeV*

3. An electron has a speed of 600 m/s with an accuracy of 0.005 %. Find the certainty with which the position of the electron. *Ans: approx. 10^{-30}*

4. A beam of Al atoms is used to dope a semiconductor chip to set its electrical properties. If the atom's velocity is known to within 0.2 m/s, how accurately can they be positioned?

   Ans: 12 nm

5. Average lifetime of an excited atomic state is 10^{-8} sec. If the wavelength of spectral line associated with the transition from this state to ground state is 6000A, estimate the width of the spectral line. *Ans: 0.19 fm*

6. An electron moves in x direction with a speed of 3.6×10^{6} m/s. We can measure its speed to a precision of 1%. With what precision can we simultaneously measure its *x* coordinate?

   Ans : 3.2 nm

7. A charged pi meson has a rest energy of 140 MeV and a lifetime of 26 ns. Find its energy uncertainty in MeV and also as a fraction of its rest energy. *Ans : 1.8×10^{-16}*

8. Estimate the minimum velocity that would be measured for a billiard ball of mass 100 g confined to a billiard table of dimension 1 m. *Ans: 10^{-33} m/s*

9. A woman on a ladder drops small pellets toward a spot on the floor. (a) Show that, according to the uncertainty principle, the 'missed' distance must be at least

$$\Delta x = (2\,\hbar / m)^{1/2}\,(2H / g)^{1/4}$$

where H is the initial height of each pellet above the floor and m is the mass of each pellet. (b) If H = 2.0 meters and m = 0.50 g, what is Δx?

10. A beam of electrons is incident on a slit of variable width. If it is possible to resolve a 1% difference in momentum, what slit width would be necessary to resolve the interference pattern of the electrons if their kinetic energy is (a) 0.010 MeV, (b) 1.0 MeV, and (c) 100 MeV ?

Unit III

1. A particle limited to the x axis has the wave function

$$\psi = ax \ \ ; \text{between } x = 0 \text{ and } x = 1$$

$\psi = 0$; elsewhere.

Find the probability that the particle can be found between x= 0.45 and x= 0.55.

Ans: 0.0251 a^2

2. Operate $\frac{d^2}{dx^2}$, kinetic energy operator, momentum operator on the given wave-function. Does these operators form Eigen value equation? If yes, find eigen value for the following wave function:

$$\psi = e^{2x}.$$

Ans: 4, $-2(h/2\pi)^2/m$, $-2i\,(h/2\pi)$

3. Which of the following ψ can't be the solution of Schrodinger equation for all values of x?

(a) A sec x (b) A tan x (c) Ae^{x^2} (d) Ae^{-x^2}

Ans: a,b,c

4. If $\psi(x) = \frac{N}{x^2 + a^2}$, calculate the normalization constant N.

$$Ans:\ N = \left(\frac{2a^3}{\pi}\right)^{1/2}$$

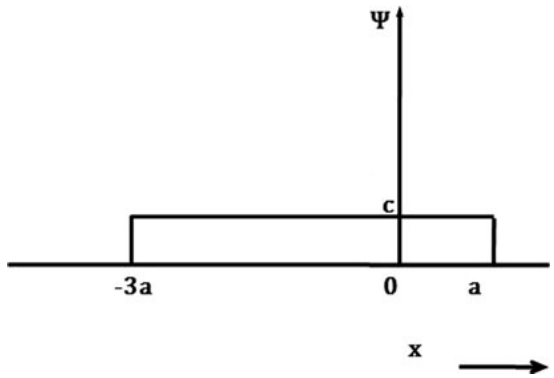
5. The state of a free particle is described by the following wave function

$$\psi(x) = 0 \text{ for } x < -3a$$

$$\psi(x) = c \text{ for } -3a < x < a$$

$$\psi(x) = 0 \text{ for } x > a.$$

(a) Determine c using the normalization condition. (b) Find the probability of finding the particle in the interval [0, a].



Ans: c=1/(2√a), P=1/4.

Unit IV

1. An electron is trapped in a one-dimensional region of length 1 A° (a typical atomic diameter). a) Find the energy of the ground state and the first two excited States. b) How much energy must be supplied to excite the electron from the ground state to the second excited state? c) From the second excited state, Electron drops down to the first excited state. How much energy is released in this process? *Ans: 37.6 eV, 300.8 eV, 188.0 eV.*

2. A particle is represented by the following wave function :

$$\psi(x) = 0, x < \frac{-L}{2}, x > \frac{L}{2},$$
$$\psi(x) = C\left(\frac{2x}{L} + 1\right), -\frac{L}{2} < x < 0,$$
$$\psi(x) = C\left(\frac{-2x}{L} + 1\right), 0 < x < \frac{L}{2}$$

a)Use the normalization condition to find C. b) Evaluate the probability to find the particle in an interval of width 0.010 L at x= L/4 (i.e between x=0.245L and 0.255L) No integral is necessary for this calculation. C) Evaluate the probability to find the particle between x = 0 and x=L/4. Find the average value of x and RMS value of x. *Ans:* $\sqrt{(3/L)}$

3. An electron is trapped in an infinitely deep potential well of width L = 10^6 fm. Calculate the wavelength of photon emitted from the transition $E_4 \rightarrow E_3$.

Ans: λ=4665nm.

4. A particle in the infinite square well has its initial wave function an even mixture of the first two stationary states:

$$\psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Normalize $\psi(x,0)$.

(b) Find $\psi(x,t)$ and $|\psi(x,t)|^2$.

(c) Find expectation value of x and p?

(d) Find the expectation value of H. How does it compare with E_1 and E_2.

Ans: $A=1/\sqrt{2}$, $\psi(x,t) = \frac{1}{\sqrt{a}} e^{-i\omega t}\left[\sin\left(\frac{\pi x}{a}\right) + \sin(\frac{2\pi x}{a}) e^{-3i\omega t}\right]$, $|\psi(x,t)|^2 = \frac{1}{a}\left[\sin^2\left(\frac{\pi x}{a}\right) + \sin^2\left(\frac{2\pi x}{a}\right) + 2\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi x}{a}\right)\cos 3\omega t\right]$, $\langle x \rangle = \frac{a}{2}\left[1 - \frac{32}{9\pi^2}\cos(3\omega t)\right]$, $\langle p \rangle = \frac{8\hbar}{3a}\sin(3\omega t)$, $\langle H \rangle = \frac{5\pi^2\hbar^2}{4ma^2}$, *average of* E_1 *and* E_2.

5. Calculate ground state and first excited state energy (eV) for an electron in 1D infinite potential well of 2A^o. Determine Normalized Eigen functions. What if the particle is a ball of mass 10g in 1D box of length 10 cm.

Ans: 9.5 eV, 38 eV (for electron); Energy is $3.44\times10^{-45} n^2$ eV for ball of mass 10g

6. A 0.5 kg baseball is confined between two rigid walls of a stadium that can be modeled as a box of length 100 m. Calculate the minimum speed of the baseball.

Ans: 1.039×10^{-35} m/s

7. A ruby laser emits 694.3 nm light. Assume light of this wavelength is due to a transition of an electron in a box from its n = 2 state to its n = 1 state. Find the length of the box.

Ans: 25.28 A^0

8. Find the points of maximum and minimum probability density for the nth state of a particle in a one dimensional box. Check your result for the n=2 state.

Ans: maximum probability density for L/4, 3L/4, minimum probability density 0, L/2, L

9. Which of the following functions are Eigen functions of the momentum operator $\hat{p}$. For those that are Eigen functions, what are the eigenvalues?

a) A sin(kx)
b) Asin (kx)-Acos(kx)
c) Acos(kx)+isin(kx)
d) $Ae^{ik(x-a)}$

Ans: d, k h/2π

10. An electron is described by the wave function

$$\psi(x) = \begin{cases} 0 & for\ x < 0 \\ Ce^{-x}(1-e^{-x}) & for\ x > 0 \end{cases}$$

Where x is in nm and C is a constant.

(a) Find the value of C that normalizes $\psi(x)$. (b) Where is the electron most likely to be found: that is, for what value of x is the probability for finding the electron largest? (c) Calculate <x> for this electron and compare your result with its most likely position. Comment on any differences you find.

Ans: (a) $12^{1/2}$ $nm^{-1/2}$ (b) 0.693nm (c)$< x >= \frac{13}{12} nm$ is somewhat greater than the most probable position.

11. For any eigenfunction ψ_n of the infinite square well, show that <x> = L/2 and that

$$< x >^2 = \frac{L^2}{3} - \frac{L^2}{2(n\pi)^2}$$

Where L is the well dimension Take the boundaries to be x=0 and x=L.

12. An electron has a wave function

$$\psi(x) = Ce^{-|x|/x_0}$$

Where x_0 is a constant and $C = \frac{1}{\sqrt{x_0}}$ for normalization. For this case, obtain expression for <x> and Δx in terms of x_0. Also calculate the probability that the electron will be found within a standard deviation of its average position, that is, in the range <x>-Δx to <x>+Δx, and show that this independent of x_0.

Ans: $< x >= 0$, $< x^2 >= x_0^2/2$, $\Delta x = x_0/\sqrt{2}$, *P=0.757*

13. Two copper conducting wires are separated by an insulating oxide layer (CuO). Modeling the oxide layer as a square barrier of height 10.0 eV, estimate the transmission coefficient for penetration by 7.00-eV electrons (a) if the layer thickness is 5.00 nm and (b) if the layer thickness is 1.00 nm. *Ans: (a)* $T=0.963\times10^{-38}$ *(b)* $T=0.657\times10^{-7}$

14. A 30-eV electron is incident on a square barrier of height 40 eV. (A) What is the probability that the electron tunnels through the barrier if its width is 1.0 nm? (B) What is the probability that the electron tunnels through the barrier if its width is 0.10 nm?

Ans: (A) 8.5×10^{-15} *(B) 0.039*

15. An electron has a kinetic energy of 12.0 eV. The electron is incident upon a rectangular barrier of height 20.0 eV and width 1.00 nm. If the electron absorbed all the energy of a photon of green light (with wavelength 546 nm) at the instant it reached the barrier, by what factor would the electron's probability of tunneling through the barrier increase?

Ans: (a) 0.010 3 (b) 0.990

Unit 5-7

1. What is the binding energy of the most common helium nucleus? Helium nucleus has two protons and two neutrons. Given: m(He) = 4.002603 amu.
   Ans:28.3 MeV
2. Find the minimum energy needed to break a carbon-12 nucleus into three alpha particles.
   Ans:7.27 MeV
3. Two deuterons fuse to form a triton (a nucleus of tritium, or $^3H_1$) and a proton. How much energy is liberated? *Ans:4.03 MeV*
4. The isotope ^{226}Ra undergoes a decay with a half-life of 1620 years. What is the activity of 1.0 g of ^{226}Ra? Express your answer in Bq and in Ci. *Ans:* 3.62×10^{10} *Bq, 0.978 Ci*

5. The half life of Radium is 1600 years. After how many years 25% of radium block remains undecayed. *Ans : 3200 years*

6. The half life of Radium is 1500 years. After how many years will 1g of pure radium be reduced to 1 cg. By what time its activity shall reduce to 75% ? *Ans : 9972 years*

7. A radioactive isotope has a half life of 5 years. How long will it take the activity to reduce to 3.125%. *Ans : 25 years*

8. Given mass of $^{238}U_{92}$= 238.05079 amu, $^{234}Th_{90}$ = 234.04363 amu, $^{4}He_{2}$ = 4.00260 amu, $^{237}Pa_{91}$=237.03121amu, $^{1}H_{1}$ = 1.00783 amu. Calculate energy released during the alpha decay of $^{238}U_{92}$, and kinetic energy of alpha particles. Further show that $^{238}U_{92}$ can't spontaneously emit a proton. *Ans : 4.25 MeV, 4.18MeV, -7.68MeV*

9. Isotope of $^{12}N_{7}$ decays to an excited state of $^{12}C_{6}$ which decays to the ground state with the emission of 4.43 MeV gamma rays. Given the mass of ^{12}N is 12.018613 u. Show that the maximum energy of beta particles is 16.32 MeV.

10. A free neutron decays into proton, electron and antineutrino. Find the kinetic energy shared by electron and an antineutrino. Given m(n)=1.00898u, m(p)=1.00759 u, m(e) = 0.00055u. *Ans : 0.78MeV*

11. Tritium emits – beta particles. Calculate its end point energy. Given m($^{3}H_{1}$)=3.01695 u, m($^{3}He_{2}$)=3.01693u, m(e)=0.00055u. *Ans : 0.01562 MeV*

12. Calculate the energy of gamma rays emitted in beta decay of $^{28}Al_{13}$ Given the end point energy =2.81MeV, Given m ($^{28}Al_{13}$)=27.9819 u, m($^{28}Si_{14}$)=27.9769 u. *Ans : 1.84MeV*

13. ^{23}Mg emits positrons. Represent the decay process by an equation and calculate the end point energy of the positron emitted. Given m(e)=0.00055u, m(^{23}Na)=22.99618u, m(^{23}Mg)= 23.0002 u. *Ans : 2.72 MeV*

14. The half life of Au(198) is 2.70 days.

    A) What is the decay constant of Au?

    B) What is the probability that any Au nucleus will decay in 1 sec?

    C) Suppose we have a 100 μg sample of 198 Au. What is its activity? How many decays per second occur when the sample is one week old?

Ans: 2.97 μs, 2.97 μ, 0.244 Curie, 1.5×10⁹ Bq

15. Three different rock samples have the ratio of numbers of ^{238}U to ^{206}Pb atoms of 0.5, 1.0, 2.0. Compute the ages of the three rocks. Given half life of ^{238}U is 4.5×10^9 years

Ans : 7.1×10^9, 4.5×10^9, 2.6×10^9 years

16. A sample of CO_2 gas from the atmosphere fills a vessel of volume 200 cm^3 to a pressure of $2.00*10^4$ Pa at a temperature 295 K. Assume that all of ^{14}C beta decays were counted, how many counts would be accumulated in one week? B) An old sample of wood is burned, and the resulting product is placed in an identical vessel at the same pressure and temperature. After one week, 1420 counts have been accumulated. What is the age of the sample Given half life of ^{14}C is 5730 years?

Ans: 9.82×10^{20} atoms, 3920 years