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Assignment

B.Sc (H) Physics VI Sem

Advanced Mathematical Physics-2

- ① For Poisson Bracket, prove the following Jacobi's identity:

$$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0$$

- ② Prove the following relation b/w Poisson Bracket and Lagrange Bracket:

$$\sum_{r=1}^{2n} [u_r, u_i] \{u_r, u_j\} = \delta_{ij}$$

- ③ A particle of mass 'm' is tied to one end of a massless spring (spring constant k & unstretched length  $r_0$ ). The other end of the spring is fixed to a point P on a smooth horizontal plane on which this particle is moving. If the instantaneous position of this particle is  $(r, \theta)$  then obtain the Lagrangian & Hamiltonian of the system. Also find equations of motion of the system.

- ④ Prove that the equation of the shortest path between two points on the surface of

(i) a sphere of radius 'a' is given by:

$$\cot \theta = \alpha \cos(\phi - \beta); \quad 0 \leq \theta \leq \pi$$

$$0 \leq \phi \leq 2\pi$$

$\alpha, \beta \rightarrow \text{constants}$

(ii) a right circular cylinder of radius 'a' is given by:  $z = c_1 \phi + c_2$ ; where  $c_1, c_2 \rightarrow \text{constants}$ .

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5 Find the Hamiltonian corresponding to the Lagrangian:

$$L = ax^2 + by^2 - kxy \quad (a, b, k \rightarrow \text{constants})$$

6 Consider the set of matrices  $G$ :

$$G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in \mathbb{Q} \text{ (set of rational numbers)} \right\}$$

Show that  $G$  forms an abelian group wrt. matrix multiplication. (Assume that matrix multiplication is associative).

Also find the center of  $G$ .

7 Show that the set of all  $2 \times 2$  matrices over real numbers having determinant  $+1$  form a group wrt. matrix multiplication.

(Assume that matrix multiplication is associative)

8 Prove that the set  $\mathbb{Q}_-$  of all rational numbers other than  $-1$  with the binary operation  $*$  defined by:

$$a * b = a + b + ab$$

form a group.

9 (i) Find all the right cosets of  $H$  in  $G$  where

$$G = \langle a \rangle \text{ of order } 12$$

$$H = \langle a^3 \rangle$$

(ii) Find all the left cosets of  $(H, +)$  in  $(G, +)$ , where

$$G = \mathbb{Z} \text{ \& } H = \{5x : x \in \mathbb{Z}\}$$

10 (i) What do you understand by Conditional Probability? Explain.

(ii) State & Prove Bayes' Theorem.