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Subject: Electricity & Magnetism
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Faraday's Law of Induction

8.1 Faraday's Law

In the previous chapter, we have shown that *steady electric current* can give *steady magnetic field* because of the symmetry between electricity & magnetism.

We can ask: *Steady magnetic field* can give *steady electric current.* ×

OR *Changing magnetic field* can give *steady electric current.* ✓

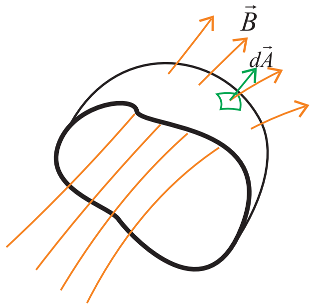
Define :

(1) Magnetic flux through surface S:

$$\boxed{\Phi_m = \int_S \vec{B} \cdot d\vec{A}}$$

Unit of Φ_m : Weber (Wb)

$1\text{Wb} = 1\text{Tm}^2$



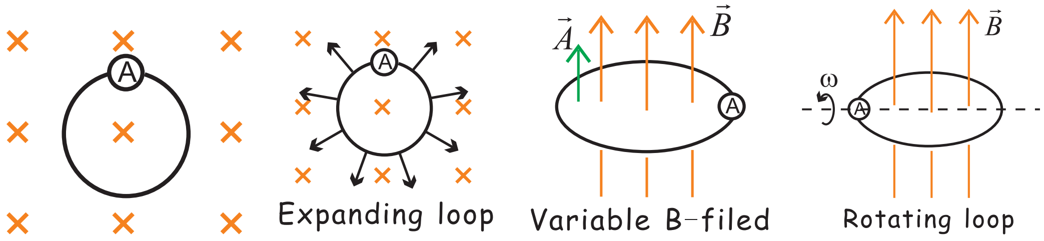
(2) Graphical:

$$\Phi_m = \text{Number of magnetic field lines passing through surface } S$$

Faraday's law of induction:

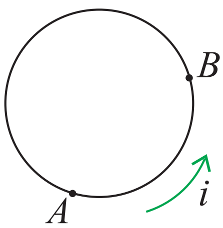
$$\boxed{\text{Induced emf } |\mathcal{E}| = N \left| \frac{d\Phi_m}{dt} \right|}$$

where N = Number of coils in the circuit.



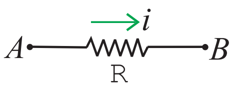
$\vec{B}$ = Constant	$\vec{B}$ = Constant	$\hat{B}$ = Constant	$\vec{B}$ = Constant
$\vec{A}$ = Constant	$\hat{A}$ = Constant	$dB/dt \neq 0$	A = Constant
	$dA/dt \neq 0$	$\vec{A}$ = Constant	$d\hat{A}/dt \neq 0$
$\mathcal{E} = 0$	$\therefore\ \lvert\mathcal{E}\rvert > 0$	$\therefore\ \lvert\mathcal{E}\rvert > 0$	$\therefore\ \lvert\mathcal{E}\rvert > 0$

Note : The *induced emf* drives a current throughout the circuit, similar to the function of a *battery*. However, the difference here is that the induced emf is *distributed throughout the circuit*. The consequence is that *we cannot define a potential difference between any two points in the circuit.*



Suppose there is an *induced current* in the loop, can we define ΔV_{AB}?

Recall:



$$\Delta V_{AB} = V_A - V_B = iR > 0$$
$$\Rightarrow\quad V_A > V_B$$

Going *anti-clockwise (same as i)*,

If we start from A, going to B, then we get $V_A > V_B$.
If we start from B, going to A, then we get $V_B > V_A$.

$$\boxed{\therefore\ \text{We cannot define } \Delta V_{AB}\ !!}$$

This situation is like when we study *the interior of a battery*.

A battery / The loop } provides the energy needed to drive the charge carriers around the circuit by { chemical reactions. / changing magnetic flux.

sources of emf — *non-electric means*

8.2 Lenz' Law

(1) The flux of the magnetic field due to induced current *opposes* the change in flux that causes the induced current.

(2) The induced current is in such a direction as to *oppose* the changes that produces it.

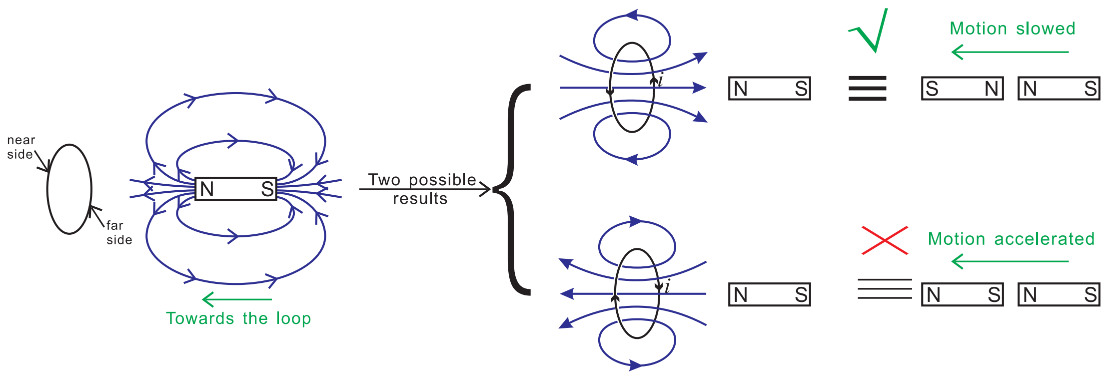
(3) Incorporating Lentz' Law into Faraday's Law:

$$\boxed{\mathcal{E} = -N\,\frac{d\Phi_m}{dt}}$$

If $\dfrac{d\Phi_m}{dt} > 0,\ \Phi_m \uparrow$ $\Rightarrow$ $\mathcal{E}$ appears $\Rightarrow$ Induced current appears.

$\Rightarrow$ $\vec{B}$-field due to induced current $\Rightarrow$ change in Φ_m $\overset{\textit{so that}}{\Longrightarrow}$ $\Phi_m \downarrow$

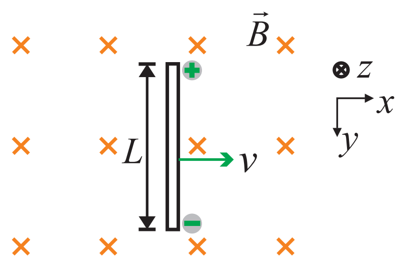
(4) Lenz' Law is a consequence from *the principle of conservation of energy.*



8.3 Motional EMF

Let's try to look at a special case when the *changing magnetic flux* is carried by *motion in the circuit wires.*

Consider a conductor of length L moving with a velocity v in a magnetic field $\vec{B}$.



Hall Effect for the charge carriers in the rod:

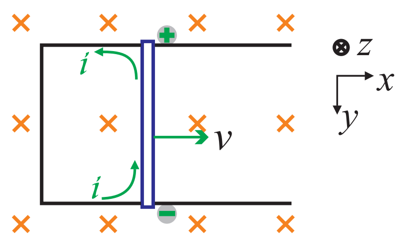
$$\begin{aligned} & \vec{F}_E + \vec{F}_B = 0 \\ \Rightarrow\ & q\vec{E} + q\vec{v} \times \vec{B} = 0 \qquad \text{(where } \vec{E} \text{ is Hall electric field)} \\ \Rightarrow\ & \vec{E} = -\vec{v} \times \vec{B} \end{aligned}$$

Hall Voltage inside rod:

$$\begin{aligned} \Delta V &= -\int_0^L \vec{E} \cdot d\vec{s} \\ \Delta V &= -EL \end{aligned}$$

$$\therefore \quad \text{Hall Voltage : } \boxed{\Delta V = vBL}$$

Now, suppose the moving wire *slides without friction* on a stationary U-shape conductor. The motional emf can drive an electric current i in the U-shape conductor.
$\Rightarrow$ Power is dissipated in the circuit.
$\Rightarrow$ $P_{out} = Vi$ (Joule's heating)
(see Lecture Notes Chapter 4)



What is the source of this power?
Look at the forces acting on the conducting rod:

- Magnetic force:

$$\begin{aligned} \vec{F}_m &= i\vec{L} \times \vec{B} \\ F_m &= iLB \quad \text{(pointing left)} \end{aligned}$$

- For the rod to continue to move at constant velocity v, we need to *apply an external force*:

$$\vec{F}_{ext} = -\vec{F}_m = iLB \quad \text{(pointing right)}$$

$\therefore$ *Power* required to keep the rod moving:

$$\begin{aligned} P_{in} &= \vec{F}_{ext} \cdot \vec{v} \\ &= iBLv \\ &= iBL\,\frac{dx}{dt} \\ &= iB\,\frac{d(xL)}{dt} \qquad (\ xL = A, \text{ area enclosed by circuit}) \\ &= i\,\frac{d(BA)}{dt} \qquad (\ BA = \Phi_m, \text{ magnetic flux}) \end{aligned}$$

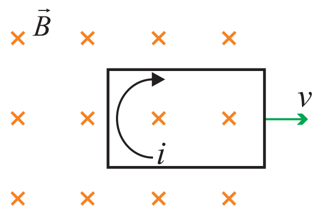
Since energy is not being stored in the system,

$$\therefore \quad P_{in} + P_{out} = 0$$
$$iV + i\frac{d\Phi_m}{dt} = 0$$

We "prove" Faraday's Law $\Rightarrow$ $\boxed{V = -\frac{d\Phi_m}{dt}}$

Applications :

(1) Eddy current: moving conductors in presence of magnetic field

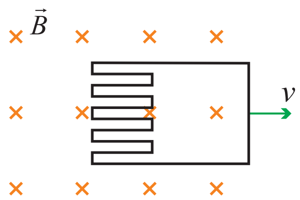


Induced current

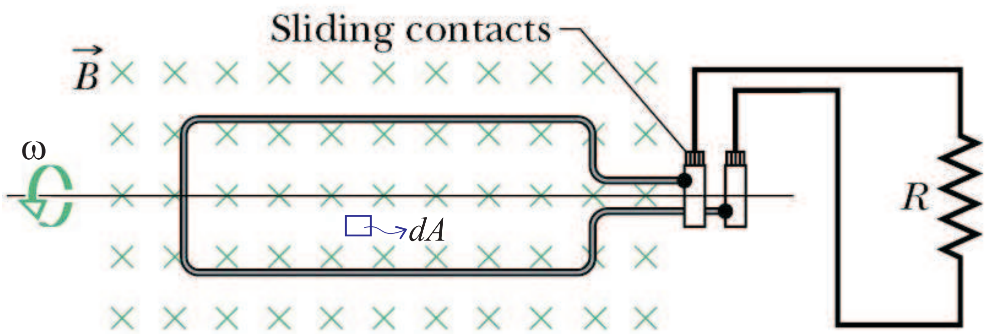
$\Rightarrow$ Power lost in Joule's heating $\left(\frac{\mathcal{E}^2}{R}\right)$

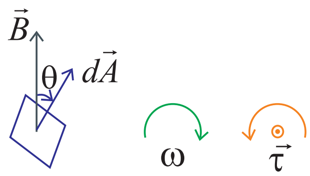
$\Rightarrow$ Extra power input to keep moving

To reduce Eddy currents:



(2) Generators and Motors:
Assume that the circuit loop is *rotating at a constant angular velocity* ω, (Source of rotation, `e.g.` steam produced by burner, water falling from a dam)





Magnetic flux through the loop

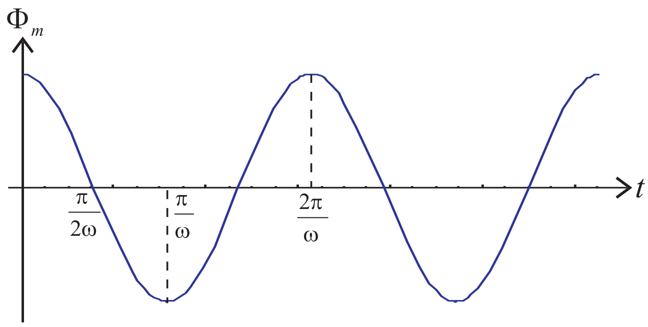
$$\Phi_B = \overset{\substack{\text{Number of coils}\\ \downarrow}}{N} \int_{loop} \vec{B} \cdot d\vec{A} = NBA \underset{\substack{\downarrow \\ \text{changes with time! } \theta = \omega t}}{\cos\theta}$$

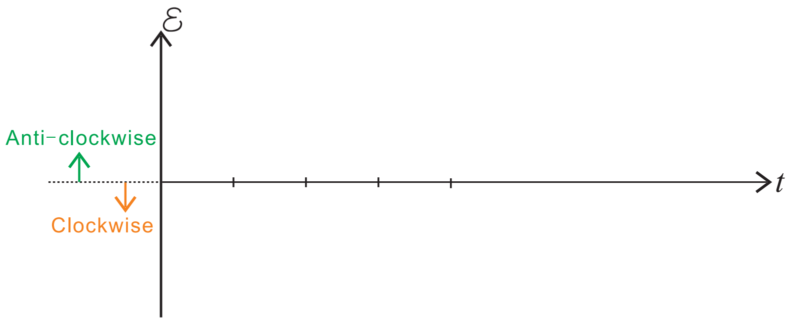
$$\therefore \quad \Phi_B = NBA\cos\omega t$$

$$\text{Induced emf:} \quad \mathcal{E} = -\frac{d\Phi_B}{dt} = -NBA\frac{d}{dt}(\cos\omega t) = NBA\omega\sin\omega t$$

$$\text{Induced current:} \quad i = \frac{\mathcal{E}}{R} = \frac{NBA\omega}{R}\sin\omega t$$

Alternating current (AC) voltage generator





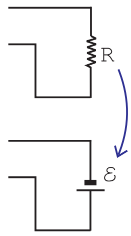
Power has to be provided by the source of rotation to overcome the torque acting on a current loop in a magnetic field.

$$\vec{\tau} = \overbrace{Ni\vec{A}}^{\vec{\mu}} \times \vec{B}$$

$$\therefore \tau = NiAB\sin\theta$$

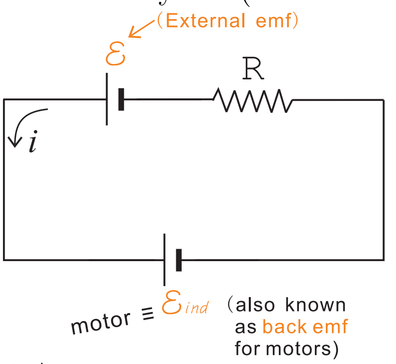
The net effect of the torque is to *oppose* the rotation of the coil.

An *electric motor* is simply a *generator operating in reverse.*
$\Rightarrow$ Replace the load resistance R with a battery of emf $\mathcal{E}$.



With the battery, there is a current in the coil, and it experiences a torque in the B-field.
$\Rightarrow$ Rotation of the coil leads to an induced emf, $\mathcal{E}_{ind}$, in the direction opposite of that of the battery. (Lenz' Law)

$$\therefore \quad i = \frac{\mathcal{E} - \mathcal{E}_{ind}}{R}$$

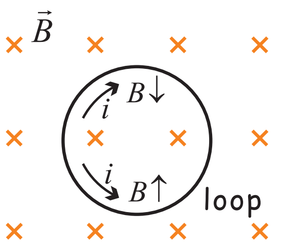


$\Rightarrow$ As motor speeds up, $\mathcal{E}_{ind}$ $\uparrow$, $\therefore$ i $\downarrow$
$\therefore$ mechanical power delivered = torque delivered = $NiAB\sin\theta$ $\downarrow$
In conclusion, we can show that

$P_{electric}$	$=$	i^2R	$+$	$P_{mechanical}$
Electric power input				Mechanical power delivered

8.4 Induced Electric Field

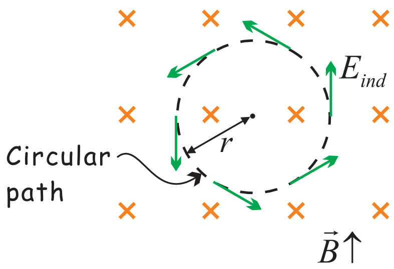
So far we have discussed that a *change* in magnetic flux will lead in an induced emf distributed in the loop, resulting from an induced E-field.



$\therefore \vec{B}\uparrow \Rightarrow$ anti-clockwise current
$\vec{B}\downarrow \Rightarrow$ clockwise current

However, even in the *absence* of the loop (so that there is no induced current), the induced E-field will still accompany a change in magnetic flux.

$\therefore$ Consider a circular path in a region with changing magnetic field.



The induced E-field only has tangential components. (i.e. radial E-field = 0)
Why?
Imagine a point charge q_0 travelling around the circular path.
Work done by induced E-field $= \underbrace{q_0 E_{ind}}_{force} \cdot \underbrace{2\pi r}_{distance}$
Recall work done also equals to $q_0\mathcal{E}$, where $\mathcal{E}$ is induced emf

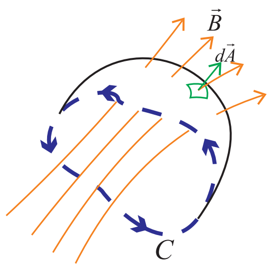
$$\therefore \quad \mathcal{E} = E_{ind} 2\pi r$$

Generally,

$$\mathcal{E} = \oint \vec{E}_{ind} \cdot d\vec{s}$$

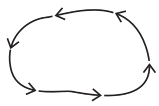
where $\oint$ is line integral around a closed loop, $\vec{E}_{ind}$ is induced E-field, $\vec{s}$ is tangential vector of path.
$\therefore$ Faraday's Law becomes

$$\boxed{\oint_C \vec{E}_{ind} \cdot d\vec{s} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{A}}$$



L.H.S. = Integral around a closed loop C
R.H.S. = Integral over a surface bounded by C

Direction of $d\vec{A}$ determined by direction of line integration C (Right-Hand Rule)

"Regular" E-field	Induced E-field
created by charges	created by changing B-field
E-field lines start from $+ve$ and end on $-ve$ charge	E-field lines form closed loops
	
can define electric potential so that we can discuss potential difference between two points	Electric potential cannot be defined (or, potential has no meaning)
$\Downarrow$ Conservative force field	$\Downarrow$ Non-conservative force field

The classification of electric and magnetic effects *depend on the frame of reference of the observer*. `e.g.` For motional emf, observer in the reference frame of the moving loop, will NOT see an induced E-field, just a "regular" E-field.
(Read: Halliday Chap.33-6, 34-7)