REFLECTION OF UNIFORM PLANE WAVES:

We have so far, studied the uniform plane waves travelling in unbounded and homogeneous media. But practically, very often, wave propagates in boundary regions consisting several media of different constitutive parameters such as $\varepsilon, \mu, \sigma, \eta$ etc.

Before we actually start with the reflection of the uniform plane wave, let us consider simple example of a transmission line.

Consider a transmission line having a characteristics impedance Z_0. Assume that the line is terminated in load impedance Z_L.

If the load impedance Z_L equals the characteristic impedance Z_0 (i.e $Z_L = Z_0$), then the line is said to be properly terminated.

If $Z_L \neq Z_0$, then there is a mismatch b/w the two impedances and the line is not properly terminated. Consider that the wave travelling along the line incidents at the load. The part of the wave gets absorbed by the load, while the other part is reflected back to the generator.

So we can say reflection occurs at the load if $Z_L \neq Z_0$. If there are two waves, one incident in forward direction, while other reflected back in backward direction, then the standing waves are said to be produced along the line.

When a uniform plane wave travels from one medium to other having different intrinsic impedances, the reflection takes place at the boundaries.

The part of the wave is transmitted in medium 2 and remaining part is reflected back to medium 1, depending upon the consecutive parameters of media.

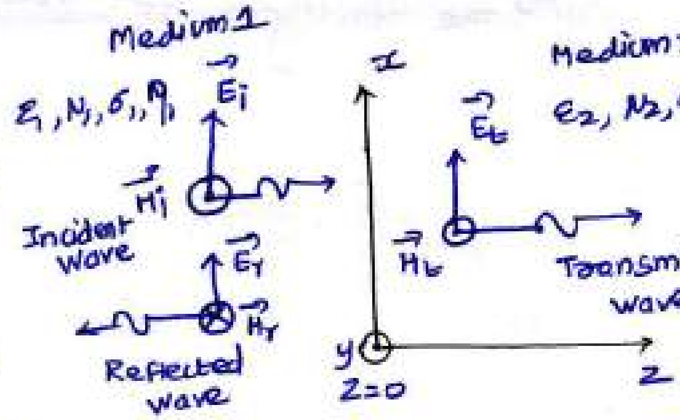
Depending upon the manner in which the uniform plane wave is incident on the boundary, there are two cases of incidence.

(i) Normal incidence

(ii) oblique incidence.

NORMAL INCIDENCE AT PLANE DIELECTRIC BOUNDARY:

Consider a uniform plane wave striking the interface b/w the two dielectrics at right angles as shown in the figure.



Assume that the uniform plane wave travels along +z direction and incidence at right angles at the boundary b/w two dielectric media i.e at $z=0$.

Below $z=0$, let the properties of medium 1 be $\varepsilon_1, \mu_1, \sigma_1, \eta_1$ and above $z=0$, the properties of medium 2 be $\varepsilon_2, \mu_2, \sigma_2, \eta_2$

So depending upon the properties of two media, part of the wave will be transmitted in medium 2 while other part will be reflected back in medium 1

Let E_i & H_i be the field strengths of the incident wave striking at the boundary.

E_t & H_t be the field strengths of the transmitted wave in the medium 2.

E_r & H_r be the field strengths of the reflected wave in the medium 1 returning back from the interface.

from figure it is clear that in medium 1, the total field composes of both the incident and reflected fields. But in medium 2 only transmitted field gives the total field.

So the conditions for the total field in medium 1 are given by,

$$\vec{E_1} = \vec{E_i} + \vec{E_r} \quad \&$$
$$\vec{H_1} = \vec{H_i} + \vec{H_r}$$

$|||^{ly}$ The conditions for total field in medium 2 is given by,

$$\vec{E_2} = \vec{E_t} \quad \&$$
$$\vec{H_2} = \vec{H_t}$$

According to the boundary condition, the tangential components of $\vec{E}$ & $\vec{H}$ must be continuous at the interface $z=0$.

$$\therefore \quad \vec{E_1}_{tan} = \vec{E_2}_{tan}$$
$$\vec{H_1}_{tan} = \vec{H_2}_{tan}$$

Thus at interface $z=0$, we can write

$$\vec{E_i} + \vec{E_r} = \vec{E_t} \quad \&$$
$$\vec{H_i} + \vec{H_r} = \vec{H_t}$$

The relationships b/w the magnitude of $\vec{E}$ & $\vec{H}$ at $z=0$ are given by the following expressions

$E_i = \eta_1 H_i$

$E_r = -\eta_1 H_r$ .... as direction of reflected wave is opposite to that of incident wave

$E_t = \eta_2 H_t$

Interms of magnitudes of the fields $\vec{E}$ & $\vec{H}$ at the interface, we can write

$$E_i + E_r = E_t \quad \& \quad \text{----} (1)$$

$$H_i + H_r = H_t \quad \text{----} (2)$$

In eqn (2), putting the values of H_i, H_r and H_t interms of E_i, E_r & E_t we get

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$$\therefore E_i - E_r = \frac{\eta_1}{\eta_2} E_t \quad \text{----} (3)$$

Adding eqn (1) & (3), we get

$$2E_i = \left(1 + \frac{\eta_1}{\eta_2}\right) E_t$$

$$2E_i = \left(\frac{\eta_1 + \eta_2}{\eta_2}\right) E_t$$

$$\boxed{E_t = \frac{2\eta_2}{\eta_1 + \eta_2} E_i} \quad \text{----} (4)$$

The Transmission co-efficient is denoted by τ and it is given by,

$$\boxed{\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}} \quad \text{----} (5)$$

Eliminating E_t from eqn (1) & (3), we get

$$\frac{(1)}{(3)} \Rightarrow \frac{E_i + E_r}{E_i - E_r} = \frac{\eta_2}{\eta_1} \Rightarrow E_i + E_r = \frac{\eta_2}{\eta_1} E_i - E_r$$

$$\eta_1 (E_i + E_r) = \eta_2 (E_i - E_r)$$

$$\eta_1 E_i + \eta_2 E_r = \eta_2 E_i - \eta_2 E_r$$

$$(\eta_1 - \eta_2) E_i = - E_r (\eta_1 + \eta_2)$$

$$E_i (\eta_1 - \eta_2) = - E_r (\eta_1 + \eta_2)$$

$$E_r = \frac{\eta_1 - \eta_2}{-(\eta_1 + \eta_2)} E_i$$

$$\boxed{E_r = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} E_i} \quad --- (6)$$

The reflection co-efficient is denoted as Γ and it is given by,

$$\boxed{\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}} \quad --- (7)$$

from eqn (5) and (7), we can draw some important results as

(a) $1 + \Gamma = \tau$

(b) $0 \leq |\Gamma| \leq 1$

(c) Both the co-efficients ; Γ and τ are dimensionless and may be complex in nature.

According to Poynting theorem, the average power density is given by,

$$P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta}\ W/m^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average power ~~transmitted~~ incident in medium-1 is given by

$$P_{iavg} = \frac{1}{2}\frac{E_i^2}{\eta_1}\ W/m^2$$

The average power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2}\frac{E_r^2}{\eta_1}\ W/m^2$$

The ratio of power transmitted to power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{\frac{1}{2}E_t^2/\eta_2}{\frac{1}{2}E_i^2/\eta_1} = \frac{\eta_1}{\eta_2}\left[\frac{E_t}{E_i}\right]^2 = \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_2+\eta_1}\right]^2 = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} P_{iavg} \quad \text{--- (1)}$$

$$= \frac{\eta_1^2+2\eta_1\eta_2+\eta_2^2-(\eta_2^2-2\eta_1\eta_2+\eta_1^2)}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1+\eta_2)^2-(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \left[\frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2} - \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}\right] P_{iavg}$$

$$\boxed{P_{tavg} = \left[1-|\Gamma|^2\right] P_{iavg}} \quad \text{--- (2)}$$

The ratio of power reflected to power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{\frac{1}{2}E_r^2/\eta_1}{\frac{1}{2}E_i^2/\eta_1} = \left[\frac{E_r}{E_i}\right]^2 = \left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right]^2 = \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} \quad \text{--- (3)}$$

Rearranging terms we get

$$P_{ravg} = \frac{(\eta_2-\eta_1)^2}{(\eta_2+\eta_1)^2} P_{iavg}.$$

$$\therefore \boxed{P_{ravg} = (\Gamma)^2 P_{iavg}} \quad --- (4)$$

Adding (1) and (3) we get

$$\frac{P_{tavg}}{P_{iavg}} + \frac{P_{ravg}}{P_{iavg}} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} + \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}$$

$$= \frac{\eta_1^2+\eta_2^2+2\eta_1\eta_2}{(\eta_1+\eta_2)^2} = \frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2}$$

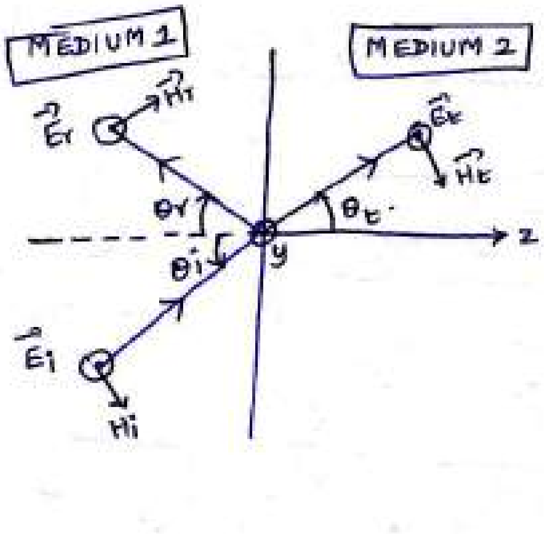
$$= 1$$

$$\therefore \boxed{P_{tavg} + P_{ravg} = P_{iavg}} \quad --- (5).$$

OBLIQUE INCIDENCE:-

When a uniform plane wave strikes obliquely on the surface (either conductor or dielectric), the behaviour of the reflected wave is decided by the polarization of the incident wave. There are two cases for the oblique incidence as given below.

case (i): The electric field vector $\perp^r$ to the plane of incidence. In other words, the Electric field vector is aligned $\parallel^{el}$ to the boundary surface as shown below. This is called Horizontal Polarization.



According to Poynting theorem, the average Power density is given by,

$$P_{avg} = \frac{1}{2}\frac{E_m^2}{\eta} \ W/m^2$$

where $E_m \rightarrow$ Amplitude of the Electric field intensity

$\eta \rightarrow$ Intrinsic impedance of a medium

The average Power ~~transmitted~~ incident in medium-1 is given by

$$P_{iavg} = \frac{1}{2}\frac{E_i^2}{\eta_1} \ W/m^2$$

The average Power reflected in medium-1 is given by

$$P_{ravg} = \frac{1}{2}\frac{E_r^2}{\eta_1} \ W/m^2$$

The ratio of Power transmitted to Power incident is given by

$$\frac{P_{tavg}}{P_{iavg}} = \frac{1/2\, E_t^2/\eta_2}{1/2\, E_i^2/\eta_1} = \frac{\eta_1}{\eta_2}\left[\frac{E_t}{E_i}\right]^2 = \frac{\eta_1}{\eta_2}\left[\frac{2\eta_2}{\eta_2+\eta_1}\right]^2 = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2}$$

Arranging terms we can write,

$$P_{tavg} = \frac{4\eta_1\eta_2}{(\eta_1+\eta_2)^2} P_{iavg} \quad \text{--- (1)}$$

$$= \frac{\eta_1^2 + 2\eta_1\eta_2 + \eta_2^2 - (\eta_2^2 - 2\eta_1\eta_2 + \eta_1^2)}{(\eta_1+\eta_2)^2} P_{iavg}$$

$$= \frac{(\eta_1+\eta_2)^2 - (\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} P_{iavg}$$

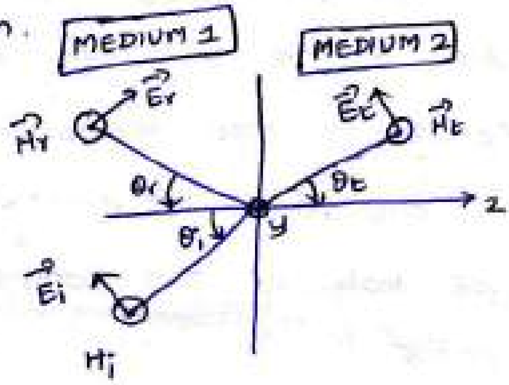
$$= \left[\frac{(\eta_1+\eta_2)^2}{(\eta_1+\eta_2)^2} - \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2}\right] P_{iavg}$$

$$\boxed{P_{tavg} = \left[1-|\Gamma|^2\right] P_{iavg}} \quad \text{--- (2)}$$

The ratio of Power reflected to Power incident is given by,

$$\frac{P_{ravg}}{P_{iavg}} = \frac{1/2\, E_r^2/\eta_1}{1/2\, E_i^2/\eta_1} = \left[\frac{E_r}{E_i}\right]^2 = \left[\frac{\eta_2-\eta_1}{\eta_2+\eta_1}\right]^2 = \frac{(\eta_2-\eta_1)^2}{(\eta_1+\eta_2)^2} \quad \text{--- (3)}$$

CASE (ii) : The magnetic field vector is aligned $\parallel^{el}$ to the boundary surface. In other words, the magnetic field vector is $\perp r$ to the plane of incidence while electric field vector is aligned $\parallel^{el}$ to the plane of incidence as shown below. This is called vertical polarization.



PLANE OF INCIDENCE:-

A plane of incidence is a plane containing the vector in the direction of propagation of the incident wave and the normal to the boundary surface.

POLARIZATION OF ELECTROMAGNETIC WAVES:

The polarization of uniform plane waves is defined as time varying behaviour of the electric field intensity vector $\vec{E}$ at some fixed point in space, along the direction of propagation.

There are three different types of polarization of a uniform plane wave as given below

(a) Linear Polarization
(b) Elliptical Polarization
(c) Circular Polarization

In other words polarization is nothing but the way in which the magnitude and direction of the electric field varies.

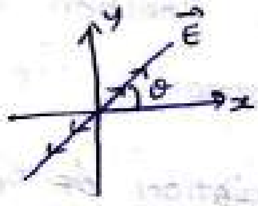
LINEAR POLARIZATION:

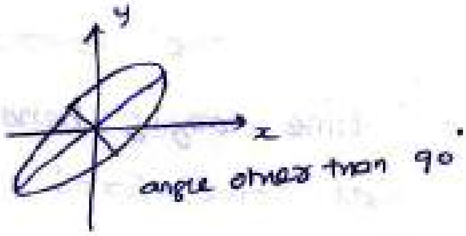
Let the components of $\vec{E}$ be $\vec{E_x}$ and $\vec{E_y}$ along x and y-direction respectively. Both these components are in phase having different amplitudes. As $\vec{E_x}$ and $\vec{E_y}$ are in phase they will have their amplitudes reaching max or min value simultaneously. Also if the amplitude of $\vec{E_x}$ increases or decreases the amplitude of $\vec{E_y}$ also increases or decreases.

In other words, at any point along +ve z-axis the ratio of amplitudes of both of the components is constant as both of them are in phase having same wavelength

The electric field $\vec{E}$ is the resultant of $\vec{E_x}$ & $\vec{E_y}$ and the direction of it depends on the relative magnitude of $\vec{E_x}$ & $\vec{E_y}$. Thus the angle made by $\vec{E}$ with x-axis is given by,

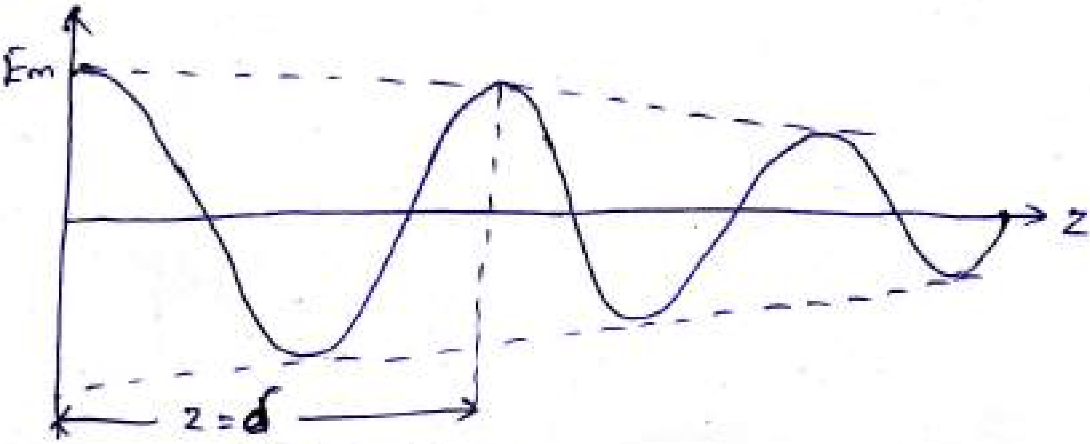
$$\theta = \tan^{-1} \frac{E_y}{E_x}.$$





SKIN DEPTH:

The skin depth is defined as the depth in which the wave has attenuated to $1/e$ ie, approximately 37% of its original value. It is also called as depth of penetration.



δ = Skin depth

Problem:

(1) A 300 Hz uniform plane wave propogate through fresh water for which sigma = 0, $\mu_r = 1$, $\epsilon_r = 78$. calculate wavelength.

Given $f = 300$ Hz, sigma = 0, $\mu_r = 1$, $\epsilon_r = 78$

$$\beta = \omega\sqrt{\mu \epsilon} = \omega\sqrt{(\mu_0\mu_r)(\epsilon_0\epsilon_r)}$$

$$\omega = 2\pi f$$

$$\beta = (2 \times \pi \times 300) \times \sqrt{(4\pi \times 10^{-7})(1)(8.854 \times 10^{-12})(78)}$$

$$\cancel{\boxed{\beta = 5.54 \times 10^{-4}\ rad/m}}$$

$$\boxed{\beta = 5.54 \times 10^{-4}\ rad/m}$$

$$\cancel{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54 \times 10^{-4}}$$

$$\boxed{\lambda = 1.13 \times 10^{-4}\ \text{metre}}$$