Principal of operation:

The reason why the rotor of an induction motor is set in to rotation is as follow,

When the 3-phase stator winding are fed by a 3-phase supply, then as said above, a magnetic flux (field) of constant magnitude but rotating at constant speed ($N_S = \frac{120f}{P}$) called synchronous speed is set up. The flux (field) passes through the air gap sweeps across the rotor ~~conductor~~ surface and cuts the rotor conductor which as yet are stationary. Due to the relative speed between the rotating flux and the stationary conductor, an emf is induced in the letter according to Faraday's laws of electromagnetic induction. The frequency of the induced emf is same as supply freq, Its magnitude is proportional to the relative velocity between the flux and the conductor. And its dirn is as given by Fleming's Right ~~left~~ hand rule. Since the rotor bar or conductor or conductors form a closed ckt, rotor current is produced, whose dirn as given by Lenz's law is such as to oppose ~~produce~~ the very cause producing it. In this case, the cause which produce

The rotor current is the relative velocity between the rotating flux of the stator and the stationary rotor conductor. Hence to reduce the relative speed, the rotor starts running in the same dirn as that of the flux and tries to catch up with rotating flux.

Note
stator field which is assured to be rotating clockwise. The relative motion of the rotor w.r.t stator is anticlockwise. By applying Right hand rule, the dirn of the induced emf is found. Now by applying Left hand rule it is clear that rotor conductor experience a force tending to rotate them in clockwise dirn. Hence the rotor set in to rotation in the same dirn as that of stator.

Slip and speed.

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In practice, the rotor never succeeds in catching up with the stator field. Let us consider for a moment that is rotor is rotating at synchronous speed. Under this condition there would be no cutting of flux by the rotor conductors, and there would be no generated voltage, no current and no torque. The rotor speed is therefore slightly less than the synchronous speed. An induction motor may also be called as "Asynchronous Motor" as it ~~depend~~ does not run at synchronous speed.

The difference between the synchronous speed and the actual rotor speed is called the slip speed ~~or slip~~.

Thus the slip speed expresses the speed of the rotor relative to the field.

If N_s — synchronous speed in r.p.m

N — actual rotor speed in rpm

then slip speed $= N_s - N$ obviously rotor (motor) speed

$\Rightarrow S = (N_s - N)/N_s$

$\%$ slip $= \dfrac{N_s - N}{N_s} \times 100$ $\Rightarrow = N = N_s(1-S)$

Actually the ~~sw~~ term slip is descriptive of the way in which rotor slip back from synchronism.

[Note: revolving flux is rotating synchronously, relative to the stator but at slip speed relative to the rotor]

The slip speed expressed as a fraction of the synchronous speed is called the unit slip. The fractional slip is usually called slip

Frequency of Rotor current

When the motor (rotor) is stationary, the frequency of rotor current is same as the supply frequency. But when the rotor starts revolving, then its frequency depend upon the relative speed (i.e difference between the synchronous speed and the rotor speed). Let at any slip speed the rotor frequency is given by f'

Then
$$N_S - N = \frac{120f'}{P} \quad -①$$

Also
$$N_S = \frac{120f}{P} \quad -②$$

Dividing ① by ② we get

$$\frac{f'}{f} = \frac{N_S - N}{N_S} = S$$

$$\Rightarrow f' = Sf$$ (the rotor freq is referred to slip)

i.e rotor current freq = per unit slip × supply freq.

① when the rotor is stationary $N = 0 \Rightarrow S = \frac{N_S - N}{N_S} = 1 \Rightarrow f' = f$

② when the rotor is driven at synchronous speed N_S then $S = 0 \Rightarrow f' = 0$

Therefore, frequency of rotor current varies from $f' = f$ at stand still $(S=1)$ to $f' = 0$ at synchronous speed

Q what will be rotor freq at the time of starting and of running of a 3-φ induction motor

Relation between Torque and Rotor P. F

In case of dc motor, the torque T_a is proportional to product of armature current and flux per pole

i.e $T_a \propto \phi I_a$

Similarly in Induction motor, the torque is also proportional to the product of flux per stator pole and rotor current. However there is other factor to be taken in to account i.e power factor of rotor.

i.e $T \propto \phi I_2 \cos\phi_2 = k \phi I_2 \cos\phi_2$

where I_2 rotor current at standstill

ϕ_2 — angle between rotor emf and rotor current

k — constt.

Denoting rotor emf at standstill by E_2, we have $E_2 \propto \phi$

$\therefore T \propto E_2 I_2 \cos\phi_2$

or $T = k_1 E_2 I_2 \cos\phi_2$ where k_1 is another constt.

continue at page 2

Starting torque:

Let E_2 = rotor emf per phase at stand still

R_2 — rotor resistance / phase.

X_2 — rotor reactance / phase at stand still

Z_2 — total impedence / phase at stand still

$= \sqrt{R_2^2 + X_2^2}$

then $I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2+X_2^2}}$

$$\Rightarrow \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2+X_2^2}}$$

Fig ①

[From impedance triangle Fig ①]

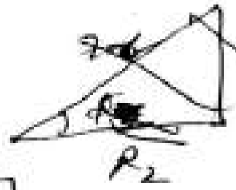

Fig ①

The standstill or starting torque.

$$T_s = K_1 E_2 I_2 \cos\phi_2$$

$$= K_1 E_2 \frac{E_2}{\sqrt{R^2+X_2^2})} \times \frac{R_2}{\sqrt{R_2^2+X_2^2}}$$

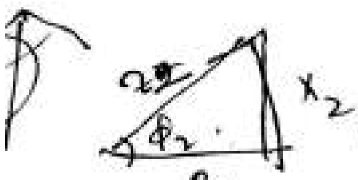

Fig ①

$$= \frac{K_1 E_2^2 R_2}{R_2^2+X_2^2} \quad \text{— eqn ②}$$

If supply voltage is constt. then the flux ϕ and hence E_2 both are constt.

$$\therefore T_{st} = \frac{K_2 R_2}{R_2^2+X_2^2}$$ where K_2 is some other constt.

Now $K_1 = \frac{3}{2\pi N_s}$

[as in DC motor $K_1 = \frac{1}{2\pi N}$ since induction motor is 3phase so $K_1 = \frac{3}{2\pi N}$

eqn ②

$$\therefore T_s = \frac{3}{2\pi N_s} \frac{E_2^2 R_2}{R_2^2+X_2^2}.$$

where N_s — synchronous speed in RPS

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Condition for max. starting torque.

To get the max value differential w.r.t R_2 and equate it zero

$$\therefore \frac{dT_{st}}{dR_2} = 0 \qquad \left[T_s = \frac{K_2 R_2}{R_2^2 + X_2^2} \right]$$

$$\Rightarrow K_2 \left[\frac{1}{R_2^2 + X_2^2} - \frac{R_2(2R_2)}{(R_2^2 + X_2^2)^2} \right] = 0$$

$$\text{or} \quad R_2^2 + X_2^2 - 2R_2^2 = 0$$

$$\Rightarrow R_2^2 + X_2^2 = 2R_2^2$$

$$\Rightarrow X_2^2 = R_2^2$$

i.e when rotor resistance = rotor reactance.

Effect of change of supply voltage on starting Torque

$$\text{Now} \quad T_{st} = \frac{K_1 E_1^2 R_2}{\sqrt{R_2^2 + X_2^2}} \quad . \quad \text{Now } E_1 \propto V$$

$$\Rightarrow T_{st} = \frac{K_3 V^2 R_2}{\sqrt{R_2^2 + X_2^2}} = \frac{K_3 V^2 R_2}{Z_2^2}$$

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(b) Rotor E.M.F and Reactance under Running Condition

under Running condition.

Let f' = Rotor current frequency.

X_r - Rotor Reactance / phase

E_r - Rotor emf / phase

Z_r - Rotor impedance / phase

R_2 - Rotor resistence / phase (remain same under both condition)

we have proved that

$f' = sf$ where s is slip

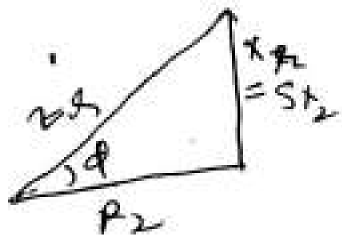
similarly $E_r = sE_2$

$X_r = sX_2$

$Z_r = \sqrt{R_2^2 + X_r^2} = \sqrt{R_2^2 + s^2X_2^2}$

Now $T \propto E_r I_r \cos\phi_2$

or $T \propto \phi I_r \cos\phi_2$



Now $E_r = sE_2$

$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{R_2^2+(sX_2)^2}}$

∴ $\cos\phi_2 = \frac{R_2}{\sqrt{R_2^2+(sX_2)^2}}$

$T \propto \frac{s\phi E_2 R_2}{R_2^2+(sX_2)^2}$; ~~[$E_2 \propto \phi$]~~ as

$= \frac{K\phi\, s E_2 R_2}{R_2^2+(sX_2)^2} = \cancel{\frac{K s E_2 R_2}{R_2^2+(sX_2)^2}}$ [$E_2 \propto \phi$

Also $T = \frac{K_1 \, s E_2^2 R_2}{R_2^2 + (s x_2)^2}$ $[\because E_2 \propto \phi]$ —(1)

At standstill $s = 1$

$$T_{st} = \frac{K_1 E_2^2 R_2}{R_2^2 + x_2^2}$$

Torque when the motor is running is given by expression (1) is maximum

when $\frac{s R_2}{R_2^2 + s^2 x_2^2}$ or $\frac{R_2}{\frac{R_2^2}{s} + s x_2^2}$ is min

or differentiate w.r.t s and equate it to zero.

we get $R_2^2 = s^2 x_2^2$

we get $\Rightarrow R_2 = s x_2$

$s = \frac{R_2}{x_2}$

$\Rightarrow R_2 = s x_2$